in the throat are the same between cold and hot operating conditions, it is evident that a close check must be made on the range of operating conditions that will give a valid determination of the stagnation enthalpy level in a particular facility

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Temperature Recovery Factors for Flow Longitudinal to a Circular Cylinder

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THE axisymmetric flow longitudinal to the exterior of a circular cylinder differs from the flow along a flat plate due to the transverse curvature of the cylindrical surface The extent to which the velocity field about the cylinder deviates from that for the flat plate has been analyzed by Seban and Bond,[1] who also investigated the corresponding heat-transfer problem for a gas with a Prandtl number of 0 715 The Seban-Bond analysis yields expressions for skin friction, heat transfer, and recovery factor in the form of series; the first term of each series corresponds to the flat-plate problem, and succeeding terms give the deviations due to transverse curvature Quantitative results were obtained from numerical solutions of a set of ordinary differential equations of the boundary-layer type An improvement in the accuracy of the velocity solutions was later achieved by Kelly,[2] who made use of an analog computer in lieu of desk calculators employed by the original investigators Within the knowledge of the present authors, the thermal results reported by Seban and Bond still stand without re-examination

It is the purpose of this investigation to re-examine the recovery-factor result of Seban and Bond and also to provide recovery-factor information for a wide range of gas Prandtl numbers The need for results encompassing a more extensive Prandtl-number range is highlighted by such applications as the recovery-factor method[3] of measuring the Prandtl number In the forthcoming presentation, the nomenclature of the Seban-Bond paper will be employed whenever possible to facilitate a concise description

One may begin by relating the recovery factor R to the dimensionless boundary-layer temperature distribution $g(\xi,\eta) = (T - T_\infty)/(U_\infty^2/c_p)$ Under the adiabatic-wall condition, the surface temperature of the cylinder may be designated as T_{aw} and the recovery factor written as

$$R = (T_{aw} - T_\infty)/(U_\infty^2/2c_p) = 2g(\xi,0) \quad (1)$$

The temperature distribution g is expressible as a perturbation series

$$g(\xi,\eta) = g_0(\eta) + \xi g_1(\eta) + \xi^2 g_2(\eta) + \quad (2)$$

Received November 19, 1963

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Table 1 Wall derivatives of the velocity function f

	Seban-Bond	Kelly	Present
$f_1''(0)$	0 704	0 696	0 694322
$-f_2''(0)$	0 095	0.160	0 164144

wherein the coordinates ξ, η are defined in terms of the radial coordinate r, the axial coordinate x, the cylinder radius a, and other standard symbols as

$$\xi = (4/a)\,(\nu x/U_\infty)^{1/2}$$
$$\eta = (U_\infty/\nu x)^{1/2}[(r^2 - a^2)/4a] \quad (3)$$

It may be seen that η is a generalization of the Blasius similarity variable which is defined as zero at the cylinder surface $r = a$, and ξ is a stretching of x which is proportional to the ratio of the boundary-layer thickness to the cylinder radius a The transverse curvature effect vanishes as $\xi \rightarrow 0$; correspondingly, $g_0(\eta)$ can be identified as the temperature distribution for the flat-plate problem

The temperature solution is carried out by substituting the g series into the energy equation along with a corresponding series for the dimensionless stream function f When terms are grouped according to powers of ξ, there emerges a set of ordinary differential equations for the functions g_0, g_1, g_2, etc The governing equations for g_0 and g_1 are available in Ref 1, Eqs (27) and (28) For g_2, one finds

$$(1/Pr)g_2'' + f_0 g_2' - 2f_0' g_2 + (1/Pr)(\eta g_1'' + g_1') + 2f_1 g_1' - f_1' g_1 + 3f_2 g_0' + \tfrac{1}{2} f_0''(f_2'' + \eta f_1'') + \tfrac{1}{4}(f_1'')^2 = 0 \quad (4)$$

where the primes denote differentiation with respect to η, and f_0, f_1, and f_2 are functions which appear in the series for f The boundary conditions appropriate to the adiabatic wall problem are $g_i'(0) = 0$ and $g_i(\infty) = 0$ for all i

To proceed, it is necessary that numerical solutions be performed These were carried out to high accuracy on a Control Data 1604 digital computer The differential equations involved in the problem are linear except for the f_0 (Blasius) equation Consequently, it was possible to construct solutions of each equation by linearly combining pairs of trial functions that satisfied the differential equation and the boundary conditions at $\eta = 0$

In order to obtain highly accurate temperature solutions, it was initially necessary to solve for the velocity functions f_0, f_1, and f_2 The quantities that essentially characterize these solutions are $f_0''(0)$, $f_1''(0)$, and $f_2''(0)$ These same quantities appear directly in the skin-friction expression The numerical value of $f_0''(0) = 1\ 32823$ is immediately recognizable as corresponding to the Blasius solution The values of $f_1''(0)$ and $f_2''(0)$ are listed in Table 1 to facilitate comparison with the prior investigations, Refs 1 and 2 It is seen from the table that the present velocity solutions represent an additional refinement relative to Kelly's recalculation of the Seban-Bond results

The quantities that are characteristic of the temperature solutions are $g_0(0)$, $g_1(0)$, $g_2(0)$, By applying Eqs (1) and (2), it follows that these same quantities appear in the recovery-factor expression

$$R/R_{fp} = 1 + \xi[g_1(0)/g_0(0)] + \xi^2[g_2(0)/g_0(0)] + \quad (5)$$

in which R_{fp}, the recovery factor for the flat plate, is given by

$$R_{fp} = 2g_0(0) \quad (5a)$$

The departures of R/R_{fp} from unity are a direct measure of the effect of transverse curvature on the recovery factor

Numerical values of the $g(0)$ quantities are listed in Table 2 for the Prandtl number range 0 6 to 1 1 Inspection of the table reveals several interesting trends First of all, it appears that the transverse curvature effect vanishes when the Prandtl number is unity, i e , $g_1(0) = g_2(0) = 0$ Corre-

Table 2 Wall values of the temperature function g

Pr	$g_0(0)$	$g_1(0)$	$g_2(0)$
0 6	0 386432	−0 0199978	0 0108439
0 7	0 417858	−0 0146513	0 00763153
0 8	0 447011	−0 00955783	0 00480933
0 9	0 474293	−0 00468295	0 00228608
1 0	0 500000	0 000000	0 000000
1 1	0.524358	0.00451234	−0.00209281

spondingly, the recovery factor for the cylinder is identical to that for the flat plate When the Prandtl number departs from unity, the sign of the transverse curvature effect is different depending upon whether $Pr > 1$ or $Pr < 1$ For moderate values of ξ (<1), the recovery factor for the cylinder falls below that for the flat plate when $Pr < 1$, whereas the opposite trend is in evidence when $Pr > 1$ The magnitude of the transverse curvature effect increases as the Prandtl number deviates more and more from unity

It is interesting to compare the foregoing results with corresponding information in the literature From the work of Seban and Bond, it appears that the only available result is the value $g_1(0) = 0\ 0034$ for $Pr = 0\ 715$ Upon comparing with the listing of Table 1, it is seen that this $g_1(0)$ is both of opposite sign and smaller by a factor of four relative to the present results Such a disparity may be due, at least in part, to inaccuracies in the velocity profiles that were used as input in the Seban-Bond solutions

The distribution of g_0 as a function of η is identical to the flat-plate solution that may be found in standard references such as Schlichting [4] Space limitations preclude a complete presentation of the g_1 and g_2 profiles; however, representative results are shown in Fig 1 for $Pr = 0\ 6$ and 1 1 As is usual for perturbation functions, the curve for g_2 crosses the zero line one more time than does the curve for g_1 Except in the neighborhood of the surface (i e , $\eta = 0$), the profiles for Pr = 0 6 and 1 1 are of the same general form However, there are distinct differences near $\eta = 0$, and these are reflected in the adiabatic-wall temperature results

It may also be mentioned that an attempt was made to derive recovery factors at large downstream distances from the leading edge of the cylinder by applying the method of Bourne and Davies,[5] but without success

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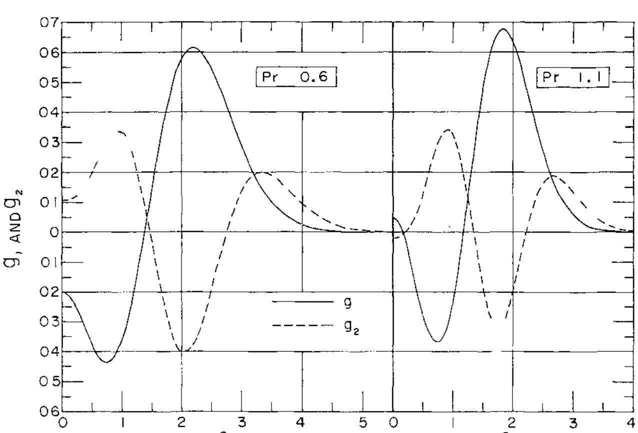

Fig 1 Representative profiles of the temperature functions g_1 and g_2

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Numerical Stability of the Three-Dimensional Method of Characteristics

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Introduction

THE numerical calculation of hyperbolic flow fields in three independent variables has been considered recently by several authors [1–4] Some fundamental differences occur in generalizing the method of characteristics from two independent variables to three For example, with two independent variables, writing the governing equations in characteristic form reduces the partial differential equations to ordinary differential equations In three independent variables the equations, even in characteristic form, are still partial differential equations

This note considers effects that arise from a second basic difference between problems in two and three independent variables In the two-dimensional case there are two characteristic lines passing through a point (neglecting the streamline), whereas in the three-dimensional case an infinite number of characteristic surfaces pass through a point (a single-parameter family) This introduces a "degree of freedom" in the choice of the net to be used in the three-dimensional problem, which is not present in the two-dimensional case This freedom of choice of the net has led to the investigation of various net configurations Fowell[1] discusses five different net configurations and the various advantages and disadvantages of each It is the purpose of this note to discuss the stability of certain of the net configurations being used at the present time

The Stability Criterion

Consider a set of first-order hyperbolic partial differential equations of the form

$$\frac{\partial u}{\partial t} = \sum_{k=1}^{m} A^k \frac{\partial u}{\partial x^k} \qquad (1)$$

where u is a vector of n unknowns, and A^k are real constant coefficient matrices Courant, Friedrichs, and Lewy[5] determined that a necessary condition for convergence of a difference scheme for this equation is that the domain of dependence of the difference scheme must contain the domain of dependence of the differential equation Hahn,[6] using the work of Lax,[7] showed that for simplicial difference schemes (i e , schemes that use a minimum number of points in the initial surface to determine a new point), the Courant-Friedrichs-Lewy condition is sufficient as well as necessary for convergence and, therefore, also for stability

Received November 18, 1963 This work was sponsored by the Air Force Office of Scientific Research under Grant AF-AFOSR 156-63 The numerical computations were carried out at the Massachusetts Institute of Technology Computation Center on an IBM 7094 computer The authors wish to gratefully acknowledge the invaluable assistance of Gilbert Strang of the Massachusetts Institute of Technology Mathematics Department who indicated the probable cause of instability and pointed out the references to the mathematical literature

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